B.SC. SIXTH SEMESTER (PROGRAMME) EXAMINATIONS, 2021

Subject: Mathematics **Course ID: 62118**

Course Code: SP/MTH/601/DSE-1B **Course Title: Mechanics**

Full Marks: 40 **Time: 2 Hours**

The figures in the margin indicate full marks

Unless otherwise mentioned the symbols have their usual meaning

1. **Answer *any five* of the following questions:** **2×5=10**

a) A system of coplanar forces is such that the sum of their moments about any two points on a given line are always same.Show that if the resultant is not zero, it must be parallel to the given line.

b) Obtain the moment of inertia of the perimeter of a circle about a tangent.

c) Find the moment of inertia of a rectangular lamina about a line through its centre and parallel to one of its edges.

d) How many independent coordinates are required for the description of a rigid body moving in two dimensions? Give reasons.

e) Find the expression for kinetic energy of a rigid body moving about a fixed axis.

f) State laws of limiting friction.

g) A particle moving in a straight line subject to a resistance which produces the retardation kv^3,where v is the velocity and$k$ is any constant.Find the velocity v and the time t in terms of the distance s.Given that initial velocity of the particle is being u.

h) Is there any difference between potential and potential energy? Justify your answer.

2. **Answer *any four*of the following questions:** **5×4=20**

a) A solid homogeneous sphere resting on the top of another fixed sphere,is slightly displaced and begins to roll down it.Show that it will slip when the common normal makes with the vertical an angle θ given by the equation $2\sin(\theta-\lambda) = 5\sin\lambda\,(3\cos\theta - 2)$, where λ is the angle of friction.

b) A sphere of weight W, is in equilibrium on a smooth plane of inclination α to the horizontal, being supported by a string, which is of length equal to the radius and it fastened to two points, one on the sphere and one on the plane. Prove that the tension of the string is $\frac{2}{3}\sqrt{3}\,W\,sin\alpha$.

c) A uniform rod of mass m and length $2a$, can turn freely about a fixed axis; show that the least angular velocity with which it must be started from the lowest position so that it may just complete revolution is $\sqrt{\frac{3g}{a}}$, with this angular velocity time to describe an angle θ is $2\sqrt{\frac{3g}{a}}\log\tan\left(\frac{\pi}{4}+\frac{\theta}{4}\right)$.

d) Four equal heavy uniform rods, each of weight W are freely jointed to form a rhombus $ABCD$ and is suspended by the joint A . A light rod connects the middle points of AB, AD so that the rhombus cannot collapse. Prove that the thrust of the light rod is $4W \tan \alpha$, where the angleBAC is α.

e) A uniform beam AB can turn freely about its end and is in equilibrium. Find the points of its length where a blow must be applied to it so that the impulses at A may be in each case $\frac{1}{n}$ th that of the blow.

f) Prove that moment of inertia of the area bounded by $r^2 = a^2 \cos 2\theta$ about its axis is $\frac{Ma^2}{16}\left(\pi - \frac{8}{3}\right)$, where M is the mass of the area.

3. Answer *any one* of the following questions: 10×1=10

a) i) A cycloid is placed with its axis vertical and vertex downwards. Show that a particle cannot rest at any point of the curve which is higher than $2a \sin^2 \lambda$ above its lower point where λ is the angle of friction and a is the radius of the generating circle of the cycloid.

ii) Show that at an extremity of the bounding diameter of a semi-circle lamina, the principal axis makes an angle $\frac{1}{2} \tan^{-1} \frac{8}{3\pi}$ to the diameter. 5+5

b) i) Three forces act along the straight lines $x = 0, y - z = a; y = 0, z - x = a; z = 0, x - y = a$. Show that they cannot reduce to a couple. Prove also that if the system reduces to a single force its line of action must lie in the surface

$$x^2 + y^2 + z^2 - 2yz - 2zx - 2xy = a^2.$$

ii) A solid frustum of a paraboloid of revolution, of height h and latus rectum $4a$, rests with its vertex on the vertex of a paraboloid of revolution whose latus rectum is $4b$; show that the equilibrium is stable if $h < \frac{3ab}{a+b}$. 6+4

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