**B.SC. SIXTH SEMESTER (PROGRAMME) EXAMINATIONS, 2021**

**Subject: Mathematics** **Course ID: 62118**

**Course Code: SP/MTH/601/DSE-1B** **Course Title: Probability and Statistics**

**Full Marks: 40** **Time: 2 Hours**

**<u>The figures in the margin indicate full marks</u>**

**Notations and symbols have their usual meaning**

1. Answer *any **five*** of the following questions: $\mathbf{2 \times 5 = 10}$

a) State the axioms of probability.

b) Find the value of the constant $k$ such that the function $f(x)$ is a possible probability density function of a random variable $X$ where

$$f(x) = \begin{cases} kx(1-x), & \text{for } 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}.$$

**c)** If $X$ is a Poisson $\mu$-variate and $P(X=0) = P(X=1)$, then find $\mu$ and $P(X \geq 1)$.

d) Show that distribution function is continuous at any point from right.

e) In a random experiment, three coins are tossed simultaneously. Write down the sample space of the experiment. Hence compute the probability of getting two heads and a tail on the coins.

f) Determine the value of k so that the function defined by $f(x, y) = kxy, 0 < x < 1,$ $0 < y < x$ is a joint probability density function.

g) Let S = {1,3,5,7} be a given population. You are required to draw a sample of size 2. Write the possible samples under (i) SRSWR & (ii) SRSWOR with usual meanings of the abbreviations.

h) State the Central limit theorem.

2. Answer *any **four** of* the following questions: $\mathbf{4 \times 5 = 20}$

a) Suppose that the joint probability density function of the 2D random variable $(X, Y)$ is

$$f(x, y) = \begin{cases} k(3x + y), & 1 \leq x \leq 3, 0 \leq y \leq 2 \\ 0, & \text{otherwise.} \end{cases}.$$

Find (i) the value of $K$, (ii) $P(X + Y < 2)$ and (iii) marginal distributions of $X$ and $Y$.

b) (i) If $X$ and $Y$ are two random variables, then show that $\{E(X, Y)\}^2 \leq E(X^2)E(Y^2)$.

(ii) For any 2-dimensional random variable $(X, Y)$, prove that $-1 \leq \rho(X, Y) \leq 1$.

**c)** Let $m$ and $\mu_r$ respectively denotes the mean and $r$-th order central moment of a Poisson distribution. Show that

$$\mu_{r+1} = r\, m\, \mu_{r-1} + m \frac{d\mu_r}{dm}.$$

d) A radioactive source emits on an average 2.5 particles per second. Calculate the probability that 3 or more particles will be emitted in an interval of 4 seconds.

e) A random variable X has probability density function $12x^2(1-x), 0 < x < 1$. Compute $P(|X-m| \geq 2\sigma)$ and compare it with the limit given by the Tchebycheff's inequality.

f) When a statistic will be called an unbiased estimator? Prove that the sample variance is not an unbiased estimator of the population variance.

**3.** Answer *any **one*** of the following questions: $\mathbf{10 \times 1 = 10}$

a) (i) Find the mean and variance of normal distribution.

(ii) State and prove Tchebycheff's inequality. $5 + 5 = 10$

b) (i) The following are the values of the variables $x$ and .

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $X$: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| $Y$: | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |

What will be the correlation coefficient between $x$ and $y$.

(ii) If $x$ and $y$ are two positively correlated variables with variances 16 and 25, respectively. Find the value of the constant $k$ such that $(x + ky)$ and $(x + \frac{4}{5}y)$ are uncorrelated. $5 + 5 = 10$

***