

**B.SC. SIXTH SEMESTER (HONOURS) EXAMINATIONS, 2021**

**Subject: Mathematics**

**Course ID: 62117**

**Course Code: SH/MTH/604/DSE-4**

**Course Title: Bio-Mathematics**

**Full Marks: 40**

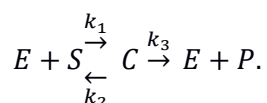
**Time: 2 Hours**

**The figures in the margin indicate full marks**

**Unless otherwise mentioned the symbols have their usual meaning**

1. Answer **any five** of the following questions: 5 × 2 = 10

- a) What assumptions are made by Malthus for formulating the population growth model?
- b) Write down Verhulst's logistic equation.
- c) What are the limitations of the Prey-Predator model.
- d) Find the fixed points of logistic map  $x_{n+1} = x_n(1 - x_n)$ .
- e) Write down the system of first order differential equations corresponding to the enzyme kinetics



- f) Write down the Routh-Hurwitz criteria of order 3.
- g) Investigate the local stability of the steady states of the system  $\frac{dx}{dt} = (x - 1)(x - 2)(x - 3)$ .
- h) Define bifurcation and bifurcation point.

2. Answer any four of the following questions: 5 × 4 = 20

- a) Find the fixed points and investigate their stability for the following logistic map  
 $x_{n+1} = r x_n(1 - x_n), \quad r > 0.$
- b) Define logistic model and interpret the model by graphical representation. 2+3
- c) Define Lotka-Volterra model. Find the fixed points of Lotka-Volterra model. Discuss geometric interpretation of the model. 1+2+2
- d) For the model equation,  $\dot{x} = f(x, y); \dot{y} = g(x, y)$ , show that  $f(x, y) = 0$  and  $g(x, y) = 0$ , pass through a common point  $(x', y')$ . If they touch each other at this point, then show that one value of eigen value of the given equation be zero and discuss the stability. 2+3
- e) Discuss Age Structured Population model and then deduce Leslie matrix. 3+2
- f) Consider the following assumptions on bacterial growth in a chemostat.  
A1) Nutrient ( $N$ ) supply to the chamber is constant ( $\gamma$ ).

A2) Bacteria ( $B$ ) growth rate ( $\beta$ ) depends linearly on consumption of the nutrient.

A3) Bacteria and nutrient are removed from the system at the rate  $\delta$ .

Based on the above assumptions write down the bacteria growth (nutrient-bacteria interaction) model and investigate the stability of the interior equilibrium point.

3. Answer any one of the following questions:

10 × 1 = 10

a) i) What is SIR model? Write down the mathematical formulation of the model. 2+3

(ii) Discuss the model for traffic on a highway and then deduce traffic wave propagation along a highway. 3+2

b) (i) Consider the difference equation

$$x_{n+1} = 0.5 x_n \text{ with } x_0 = 1024.$$

Solve the dynamical system and compute  $x_{10}$ .

(ii) Consider the Nicholson-Baily host-parasite model as

$$H_{t+1} = k H_t e^{-a P_t}$$

$$P_{t+1} = c H_t (1 - e^{-a P_t})$$

where  $H_t$  &  $P_t$  be the host and parasitoid population size at time  $t$ . Here  $a$  is the searching efficiency of the parasitoid and  $c$  be the number of viable eggs which parasitoid lays on a single host.

Find the fixed points and investigate the stability property of them.

3 + 7 = 10

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