B.SC. SIXTH SEMESTER (HONOURS) EXAMINATIONS, 2021

Subject: Mathematics **Course ID: 62117**

Course Code: SH/MTH/604/DSE-4 **Course Title: Bio-Mathematics**

Full Marks: 40 **Time: 2 Hours**

<u>The figures in the margin indicate full marks</u>

<u>Unless otherwise mentioned the symbols have their usual meaning</u>

1. Answer ***any five*** of the following questions: $5 \times 2 = 10$

   a) What assumptions are made by Malthus for formulating the population growth model?

   b) Write down Verhulst's logistic equation.

   c) What are the limitations of the Prey-Predator model.

   d) Find the fixed points of logistic map $x_{n+1} = x_n(1 - x_n)$.

   e) Write down the system of first order differential equations corresponding to the enzyme kinetics

   $$E + S \underset{k_2}{\overset{k_1}{\rightleftarrows}} C \overset{k_3}{\rightarrow} E + P.$$

   f) Write down the Routh-Hurwitz criteria of order 3.

   g) Investigate the local stability of the steady states of the system $\frac{dx}{dt} = (x-1)(x-2)(x-3)$.

   h) Define bifurcation and bifurcation point.

2. Answer any four of the following questions: $5 \times 4 = 20$

   a) Find the fixed points and investigate their stability for the following logistic map

   $x_{n+1} = r\, x_n(1 - x_n), \quad r > 0.$

   b) Define logistic model and interpret the model by graphical representation. 2+3

   c) Define Lotka-Volterra model. Find the fixed points of Lotka-Volterra model. Discuss geometric interpretation of the model. 1+2+2

   d) For the model equation, $\dot{x} = f(x,y); \dot{y} = g(x,y)$, show that $f(x,y) = 0$ and $g(x,y) = 0$, pass through a common point (x',y'). If they touch each other at this point, then show that one value of eigen value of the given equation be zero and discuss the stability. 2+3

   e) Discuss Age Structured Population model and then deduce Leslie matrix. 3+2

   f) Consider the following assumptions on bacterial growth in a chemostat.

   A1) Nutrient (N) supply to the chamber is constant (γ).

A2) Bacteria (B) growth rate (β) depends linearly on consumption of the nutrient.

A3) Bacteria and nutrient are removed from the system at the rate δ.

Based on the above assumptions write down the bacteria growth (nutrient-bacteria interaction) model and investigate the stability of the interior equilibrium point.

3. Answer any one of the following questions: $10 \times 1 = 10$

a) i) What is SIR model? Write down the mathematical formulation of the model. 2+3

(ii) Discuss the model for traffic on a highway and then deduce traffic wave propagation along a highway. 3+2

b) (i) Consider the difference equation

$x_{n+1} = 0.5\, x_n \quad with \;\; x_0 = 1024.$

Solve the dynamical system and compute x_{10}.

(ii) Consider the Nicholson-Baily host-parasite model as

$H_{t+1} = k\, H_t e^{-aP_t}$

$P_{t+1} = c\, H_t(1 - e^{-aP_t})$

where $H_t \& P_t$ be the host and parasitoid population size at time t. Here a is the searching efficiency of the parasitoid and c be the number of viable eggs which parasitoid lays on a single host.

Find the fixed points and investigate the stability property of them. $3 + 7 = 10$

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