B.SC. SIXTH SEMESTER (HONOURS) EXAMINATIONS, 2021

Subject: Mathematics **Course ID: 62116**

Course Code: SH/MTH/603/DSE-3 **Course Title: Mechanics**

Full Marks: 40 **Time: 2 Hours**

The figures in the margin indicate full marks

Unless otherwise mentioned the symbols have their usual meaning

1. Answer *any FIVE* of the following questions: 2×5=10

a) Find moment of momentum of the body about a fixed axis.

b) If a point moves so that its angular velocity about two fixed points is the same, prove that it describes a circle.

c) State the principle of conservation of linear momentum under finite forces.

d) If two particles P and Q describe the same ellipse under the same central force to the centre C, prove that the area of the triangle CPQ is invariable.

e) Define impressed forces and effective forces.

f) Deduce the work done in stretching an elastic string.

g) A particle moves along a straight line according to the law $s^2 = 6t^2 + 4t + 3$ where s is the distance at time t. Prove that the acceleration varies as $\frac{1}{s^3}$.

h) Find the moment of inertia of a door of mass m, length 2ℓ and width ℓ about its longer side.

2. Answer *any FOUR* of the following questions: 5×4=20

a) A particle is at rest on a smooth horizontal plane which commences to turn about a straight line lying in itself with constant angular velocity ω downwards; if a be the distance of the particlefrom the axis of rotation at *t=0*, show that the body will leave the plane at time t given by the equation

$$\mathrm{a}\sinh \omega t + \frac{\mathrm{g}}{2\omega^2}\cosh \omega t = \frac{\mathrm{g}}{\omega^2}\cos\omega t$$

b) Find the centre of gravity of a semi-circular plate of radius a whose mass per unit area at any point varies as $\sqrt{a^2 - r^2}$, where r is the distance of the point from the centre.

c) State and prove the principle of virtual work for a system of coplanar forces acting at different points of a rigid body.

d) If a planet were suddenly stopped in its orbit, supposed circular, show that it would fall into the Sun in a time which is $\frac{\sqrt{2}}{8}$ times the period of the planet's revolution.

e) A particle moves with a central acceleration $\mu(r + \frac{a^4}{r^3})$ being projected from an apse at distance a with a velocity$2\sqrt{\mu a}$; show that it describes the curve $r^2\left[2 + \cos\sqrt{3\theta}\right] = 3a^2$.

f) Prove that a uniform triangular lamina of mass M, and a system consisting of three particles each of mass $\frac{M}{3}$, situated at the middle point of the sides and rigidly connected by light rods, have the same moment of inertia about any axis in the plane.

3. Answer *any ONE* of the following questions: **10×1=10**

a) (i) If the resistance of the air to a particle's motion be n times its weight, and the particle be projected horizontally with velocity V, show that the velocity of the particle, when it is moving at an inclination φ to the horizontal, is $V(1 - sin\varphi)^{\frac{n-1}{2}}(1 + sin\varphi)^{\frac{n+1}{2}}$. 5

(ii) A planet is describing an ellipse about the Sun as focus; show thatits velocity away from the Sun is greatest when the radius vectorto the planet is at right angles to the major axis of the path, and that it then is $\frac{2\pi ae}{T\sqrt{1-e^2}}$, where $2a$ is the major axis, e the eccentricity, and T the periodic time. 5

b) **(i)** If the moments and product of inertia of a body (or a system of body) are given about a set of rectangular axes through O, find the moment of inertia of the system about a line OL whose direction cosines are l, m, n with respect to the rectangular axes at O. 6

ii) Find the law of force in which a particle describes an arc of a circle which passes through the centre of force. 4

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