B.Sc. 3rd Semester (Programme) Examination, 2019-20

MATHEMATICS

Course ID : 32118 **Course Code : SP/MTH-301/C-1C**

Course Title: Algebra

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Unless otherwise mentioned, notations and symbols have their usual mainly.

1. Answer *any five* questions: 2×5=10

(a) Find the value of $\left(-1+\sqrt{3}i\right)^{12}$.

(b) Transform the equation to remove the squre term from the equation $x^3+9x^2+15x-25=0$

(c) State first and second principle of induction.

(d) Define rank of a matrix and hence find the rank of $\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$.

(e) If *a, b, c* be positive real numbers, not all equal, prove that $(a+b)(b+c)(c+a)>8abc$.

(f) A relation ρ is defined on the set ℤ by $a\ \rho\ b$ if and only if $a-b$ is divisible by 5 for $a, b \in \mathbb{Z}$. Show that ρ is an equivalence relation on ℤ.

(g) Let $f, g: \mathbb{R} \to \mathbb{R}$ be defined by $f(x)=4x-1$ and $g(x)=x^2+2$. Find *fog* and *gof*.

(h) Define $T: \mathbb{R}^3 \to \mathbb{R}^2$ by $T(x,y,z)=(x-y,2z)$, show that T is linear.

2. Answer *any four* questions: 5×4=20

(a) (i) Solve the equation $16x^4-64x^3+56x^2+16x-15=0$. Whose roots are in arithmetic progression.

(ii) Using Descarte's rule of sign, find the number of complex root of the equation $2x^4+x^2+7x-6=0$. 3+2=5

(b) The matrix of a linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ with respect to the ordered basis $\{(0,1,1),(1,0,1),(1,1,0)\}$ is given by $\begin{pmatrix} 0 & 3 & 0 \\ 2 & 3 & -2 \\ 2 & -1 & 2 \end{pmatrix}$. Find T.

(c) Find the eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 1 & -1 & 2 \\ 2 & -2 & 4 \\ 3 & -3 & 6 \end{pmatrix}$.

(d) (i) Let *a, b, c, d* be positive real numbers, not all equal. Use Cauchy–Schwarz inequality to show that $(a+b+c+d)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)>16$.

(ii) If *a, b, c* be positive real numbers and $abc=k^3$, prove that $(1+a)(1+b)(1+c)\geq(1+k)^3$. 2+3=5

(e) (i) Use first principle of induction to show that $7^{2n} + 16n - 1$ is divisible by 64, $\forall n \in \mathbb{N}$.

(ii) Use theory of congruence, find the remainder when 3^{36} is divided by 77. 3+2=5

(f) Solve the system of equations

$$x + 2y + z = 1$$
$$3x + y + 2z = 3$$
$$x + 7y + 2z = 1$$

3. Answer *any one* question: 10×1=10

(a) (i) Find the remainder when 3^{36} is divided by 77.

(ii) Solve the equation using Cardan's method $x^3 - 27x - 54 = 0$.

(iii) Solve the equation $x^8 + x^7 + x^6 + \cdots + x + 1 = 0$ 2+5+3=10

(b) (i) State Cayley-Hamilton theorem.

(ii) Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$. Hence find A^{-1}.

(iii) Obtain a row-echelon matrix which is equivalent to $\begin{pmatrix} 0 & 0 & 2 & 2 & 0 \\ 1 & 3 & 2 & 4 & 1 \\ 2 & 6 & 2 & 6 & 2 \\ 3 & 9 & 1 & 10 & 6 \end{pmatrix}$.

1+(3+2)+4=10