SH-I/Physics-101C-1(T)/19

B.Sc. Semester I (Honours) Examination, 2018-19

PHYSICS

Course ID : 12411 **Course Code : SHPHS-101C-1(T)**

Course Title : Mathematical Physics I

Time: 1 Hour 15 Minutes **Full Marks: 25**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Section-I

1. Answer *any five* questions of the following: 1×5=5

(a) Find a vector perpendicular to both $\vec{A} = 2\hat{\imath} - \hat{\jmath} - 4\hat{k}$ and $\vec{B} = 3\hat{\imath} - \hat{\jmath} - \hat{k}$.

(b) Sketch $\theta =$ Constant surface in spherical polar co-ordinate system.

(c) If $\vec{E} = -\vec{\nabla}\varphi, \varphi$ is the scalar function, then find $\oint_C \vec{E} \,.\, \overrightarrow{dr}$ if C is any simple closed curve.

(d) Find the value of $\beta(\frac{1}{2}, \frac{3}{2})$.

(e) Determine the integrating factor for the differential equation $y\,dx - x\,dy = 0$.

(f) State the condition for which $x = x_0$ is a regular singular point of the differential equation $P_1(x)\frac{d^2y}{dx^2} + P_2(x)\frac{dy}{dx} + P_3(x)y = 0$.

(g) By using Rodrigue's formula for Legendre's polynomial, show that $P_2(x) = \frac{1}{2}(3x^2 - 1)$.

(h) Form the differential equation by eliminating the arbitrary function *f* and *g* from $z = f(x + at) + g(x - at)$.

Section-II

2. Answer *any two* questions of the following: 5×2=10

(a) For a solenoidal vector $\overrightarrow{A}$, show that $\vec{\nabla} \times [\vec{\nabla} \times \{\vec{\nabla} \times (\vec{\nabla} \times \vec{A})\}] = \nabla^4 \vec{A}$.

(b) Prove that $\vec{\nabla}\varphi$ points in the direction of maximum rate of increase of the scalar point function $\varphi(x, y, z)$. 3+2=5

3. Represent the vector $\vec{A} = Z\hat{\imath} - 2x\hat{\jmath} + y\vec{k}$ in cylindrical co-ordinate system. 5

4. Solve: $\left(\frac{d^2y}{dx^2} + 4y\right) = e^x \sin^2 x$. 5

5. (a) Prove that $P_n'(x) - xP_{n-1}'(x) = nP_{n-1}(x)$

(b) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. 3+2=5

Section-III

Answer *any one* question. 10×1=10

6. (a) Write down the two-dimensional Laplace's equation in polar co-ordinate.

(b) Find the general solution of the above equation by the method of separation of variables.

(c) Prove that $\beta(m,m) \times \beta\left(m+\frac{1}{2}, m+\frac{1}{2}\right) = \frac{\pi}{m}(2)^{1-4m}$. 2+5+3=10

7. (a) Evaluate $\int_C \vec{A}.\overrightarrow{dr}$ along the curve $x^2+y^2=1, z=1$ in the positive direction from (0, 1, 1) to (1, 0, 1) if $\vec{A} = (yz+2x)\hat{\imath} + xz\hat{\jmath} + (xy+2z)\hat{k}$.

(b) Find the acute angle between the surfaces $xy^2z = 3x + z^2$ and $3x^2 - y^2 + 2z = 1.$ at the point (1, –2, 1).

(c) Evaluate: div $\left(\frac{\vec{r}}{r^3}\right)$ 4+4+2=10

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